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Dynamics of Rotor Rub in Annular Clearance with Experimental Evaluation

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Rubbing phenomena in rotor dynamics are investigated by theoretical and experimental approaches. Generating conditions of forward whirling, backward rolling, backward slipping, and partial rubbing are established for system parameters and rotor speeds. Possible whirling motions of the rotor in annular clearance are calculated for full annular rub under the conditions of positive normal force and geometric constraints. The theoretical calculations show that greater clearance results in greater runout and normal force, and the whirling responses are characterized by which natural frequency of the rotor or the stator is greater than the other. Receptances of the rotor and stator are used to explain the possibility of slipping for backward slipping and forward whirling, respectively. Effect of rotor eccentricity is also considered to find the maximum rotor speed for backward slipping during increasing rotor speed and the occurrence of forward whirl. Experiments are performed for two cases. The experimental results show good agreements with the theoretical predictions.

Key Words: Rotor Dynamics, Annular Clearance, Forward Whirl, Backward Whirl, Backward Rolling, Backward Slipping, Receptance

1. Introduction

To increase the performance of rotating machinery of jet engines, steam turbines, electric motors, or centrifugal pumps, clearance between rotor and stator should be made as small as possible. This small clearance makes it difficult to avoid the contact between rotor and stator during operation. The contact between rotor and stator during whirling motion, called a rubbing, sometimes brings serious damages to rotating machinery. Depending on system parameters and rotor speed, a specific rubbing phenomenon occurs.

Rubbing phenomena can be divided into full annular rub and partial rub. Partial rub is the intermittent contact between rotor and stator. Whereas, full annular rub is the continuous contact during whole whirling motion. The whirling direction may be the same as rotor spin direction, forward whirling, or opposite, backward whirling. In the case of forward whirling the whirling frequency is usually the same as the rotor speed, called forward synchronous whirling. Forward whirl always accompanies slipping. On the other hand, backward whirl goes with slipping or no slipping depending on rotor speed. If there is no slipping at the contact surface, the rotor just rolls on the inner surface of the stator, called backward rolling. If slipping exists, it is called backward slipping. During increasing or decreasing rotor speed, different rubbing phenomena take place depending on rotor speed range, i.e., multiple solutions exist with the same operating parameters as usually shown in nonlinear problems.

Once rubbing occurs during the operation of rotating machinery, runouts of the stator and rotor become large and the operation could be destructive. Because of this disastrous phenomenon in rotating machinery, many researchers have been interested for safe and reliable design of rotating machinery. Black(1968) can be written as a notable contributor to this problem. He tried to explain the physics of rub using the polar rece-

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ptance of rotor and stator on radial symmetric rotor. Ehrich(1966) made a simple model and calculated the responses of rotor and stator numerically. Muszynska(1989) did an extensive study on this rubbing problem. Muszynska tried numerical simulations and experiments for the various cases. Numerical simulations were performed by Choy and Padovan(1987). Numerical simulation of this rubbing problem has some difficulties to calculate the contact force between rotor and stator since the contact force depends on the relative displacements of the rotor and stator. Choi and Noah(1987)) treated this problem as a piecewise linear vibration problem, and wrote an algorithm to calculate the steady state solution using FFT technique. The result showed that super and sub-harmonic responses can be found due to rubbing. Crandall et al.(1990) showed a very specific pattern of whirling frequency responses during rubbing by an experiment, and developed a theory to explain backward rolling and backward slipping.

For the purpose of physical explanation of rubbing phenomena a new approach is attempted in this study using complex notation and the receptance of rotor and stator including eccentricity effect for a flexible stator. The theoretical results are also compared with the experiments.

2. Rubbing in Annular Clearance

Figure 1 shows a schematic diagram of rubbing

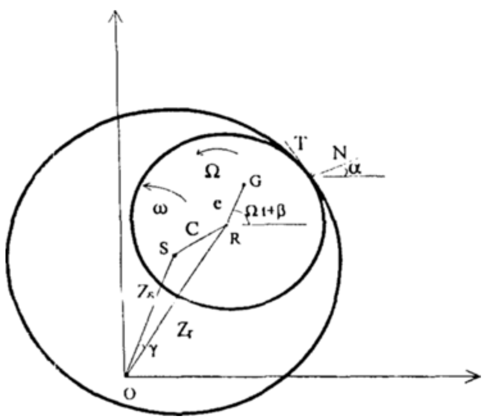

Fig. 1 Schematic diagram of rotor/stator rub

motion between rotor and stator in annular clearance. Point O represents the geometric center of both end bearings, S is the center of the stator and R is the center of the rotor during rubbing. Both rotor and stator are assumed to be linear systems. It is also assumed that the center of the rotor is perfectly aligned with the endpoints bearings. In Fig. 1, Ω is the rotor speed, and ω is the speed of the rotor and stator, i.e., whirling speed. The complex displacement Z_r of the rotor, and Z_s of the stator, are measured from the fixed point O. During rubbing, the displacement of the rotor and stator must satisfy the geometric conditions of Eq. (1) for full annular rub, and the equation of whirling motion of the rotor and stator can be written as Eq. (2).

$$Z_r - Z_s = Ce^{i\alpha} \tag{1}$$

$$M_r\ddot{Z}_r + C_r\dot{Z}_r + K_rZ_r + N(1+i\mu)e^{i\alpha} = M_r e\Omega^2 e^{i(\Omega t-\beta)}$$
$$M_s\ddot{Z}_s + C_s\dot{Z}_s + K_sZ_s + N(1+i\mu)e^{i\alpha} = 0 \tag{2}$$

Where, M_r and M_s are the rotor mass and the stator mass, respectively. C_r, C_s, K_r and K_s are the damping and the stiffness coefficients of the rotor or the stator. N represents the normal force between the rotor and stator. During rubbing the normal force should be greater than or equal to zero. μ is the ratio of the tangential force to the normal force, not to be confused with friction coefficient. If we assume the friction to be of Coulomb type, the value of μ cannot be greater than Coulomb friction coefficient of the materials of the rotor and stator. α is the angle between the normal force and the fixed x axis. Since the time derivative of a is the whirling speed, ω, there is a phase difference, τ, between the rotor and normal force in steady state responses. Relative velocity of the geometric center of the rotor with respect to the stator at the contact point should be greater than or equal to zero from physical grounds.

$$V_{rel} = a\Omega + C\omega \geq 0 \tag{3}$$

Where a is the diameter of rotor and C is the clearance. During forward whirling, the ratio of tangential force to normal force will be a kinetic friction coefficient because the relative velocity at the contact point is always greater than zero. The existence of a friction force at the contact point will oppose the relative motion of the rotor with respect to the stator. If the resistance is enough, there will be no slip between the rotor and stator. The rotor then just rolls on the inner surface of the stator. This is backward rolling with a whirling speed of $\omega = -a\Omega/C$ from Eq. (3). If there is slipping at the contact point and the whirling direction is opposite to the rotor spin direction, we have the case of backward slipping and its whirling speed will be $0 > \omega > -a\Omega/C$.

The solution of Eq. (2) along with the geometric condition (1) can explain the whirling motion of the rotor and stator. The equation of motion is composed of two linear differential equations along with a geometric requirement and a positive normal force condition. Consequently this rotor/stator system is a nonlinear system, and multiple solutions can be expected depending on system parameters and rotor speeds.

3. Elastic Stator

Neglecting the eccentricity of the rotor in Eq. (2), the following equations are resulted. These equations do not contain rotor speed term because of neglecting eccentricity.

$$
\begin{aligned}
M_r\ddot{Z}_r + C_r\dot{Z}_r + K_rZ_r + N(1+i\mu)e^{i(\omega t + \tau)} &= 0 \\
M_s\ddot{Z}_s + C_s\dot{Z}_s + K_sZ_s + N(1+i\mu)e^{i(\omega t + \tau)} &= 0
\end{aligned} \tag{4}
$$

The steady state solutions of the equation of motion will have the following forms since the normal force has the frequency component of ω.

$$
\begin{aligned}
Z_r &= r_r e^{i\omega t} \\
Z_s &= r_s e^{i(\omega t + \gamma)}
\end{aligned} \tag{5}
$$

where γ is the phase difference between the rotor and stator. Substituting Eq. (5) to the geometric condition of the full annular rub Eq. (1), yields Eq. (6).

$$
r_r - r_s e^{i\gamma} = Ce^{i\tau} \tag{6}
$$

Equation (4) and the geometric condition of Eq. (6) make six algebraic equations with the six unknowns of r_r, r_s, τ, γ, N and μ. Through manipulation of these six equations, the followings result.

$$
\begin{aligned}
&r_r = \frac{M_sC[(\omega_s^2-\omega^2)^2+(\eta_s\omega_s\omega)^2]^{1/2}}{M_{rs}[(\omega_{rs}^2-\omega^2)^2+(\eta_{rs}\omega_{rs}\omega)^2]^{1/2}},\quad r_s = \frac{M_rC[(\omega_r^2-\omega^2)^2+(\eta_r\omega_r\omega)^2]^{1/2}}{M_{rs}[(\omega_{rs}^2-\omega^2)^2+(\eta_{rs}\omega_{rs}\omega)^2]^{1/2}} \\
&N = \frac{M_rM_sC[(\omega_r^2-\omega^2)^2+(\eta_r\omega_r\omega)^2]^{1/2}[(\omega_s^2-\omega^2)^2+(\eta_s\omega_s\omega)^2]^{1/2}}{M_{rs}[(\omega_{rs}^2-\omega^2)^2+(\eta_{rs}\omega_{rs}\omega)^2]^{1/2}[1+\mu^2]^{1/2}}\cos\theta \\
&\gamma = \beta_r - \beta_s - \pi \\
&\tau = \beta_{rs} - \beta_s \\
&\theta = \beta_r + \beta_s - \beta_{rs}
\end{aligned} \tag{7}
$$

where

$$
\begin{aligned}
&\beta_r = \tan^{-1}\frac{\eta_r\omega_r\omega}{\omega_r^2-\omega^2},\quad \beta_s = \tan^{-1}\frac{\eta_s\omega_s\omega}{\omega_s^2-\omega^2},\quad \beta_{rs} = \tan^{-1}\frac{\eta_{rs}\omega_{rs}\omega}{\omega_{rs}^2-\omega^2} \\
&\tan\theta = \mu \\
&M_{rs} = M_r + M_s,\quad \omega_{rs}^2 = \frac{K_r+K_s}{M_r+M_s},\quad \eta_{rs} = \frac{C_r+C_s}{(M_r+M_s)\omega_{rs}}
\end{aligned}
$$

where rs means the combination of rotor and stator, η is a loss factor.

Assuming zero damping for both the rotor and stator, it can be known that if the rotor and stator whirls with the whirling frequency corresponding to the combined natural frequency of the rotor and stator, the normal force becomes infinite and the runouts of the rotor and stator have resonance condition. It can also be found that greater clearance results in greater normal force and greater runouts of the rotor and stator. As the whirling speed increases, the sign of the normal force changes. The pattern of changing sign is classified depending upon whether the natural frequency of the rotor or the stator is greater than the other. In this respect, the following two cases are adopted in this study.

Variations of the normal and tangential force on whirling frequencies is shown in Fig. 2 for case 1. The runouts of the rotor and stator are

Table 1 The parameters of the rotor/stator system

		Case 1	Case 2
Clearance	(mm)	1.035	0.837
Rotor natural freq.	(Hz)	11.7	16.6
Rotor loss factor		0.02	0.02
Rotor mass	(Kg)	0.526	0.260
Stator natural freq.	(Hz)	23.03	11.9
Stator loss factor		0.08	0.02
Stator mass	(Kg)	0.853	0.304

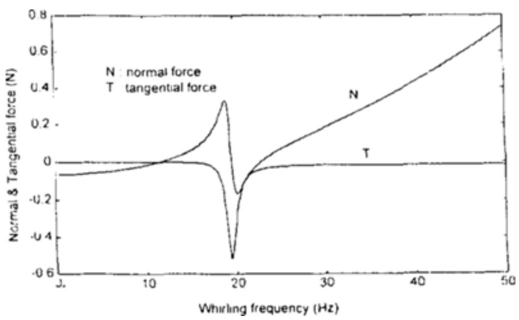

Fig. 2 Normal and tangential force vs. whirling frequency for case 1

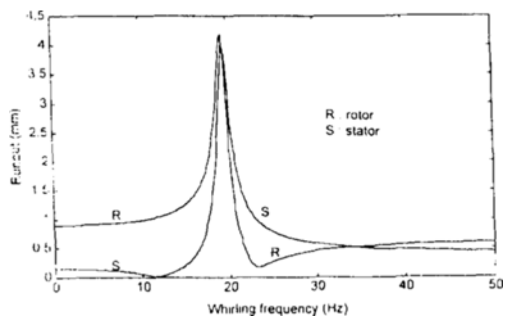

Fig. 3 Runout of rotor and stator vs. whirling frequency for case 1

shown in Fig. 3. It is evident that the whirling frequency should be $\omega_r < \omega_z < \omega_{rs}$ or $\omega > \omega_s$ to satisfy positive normal force with Eq. (7). ω_{rs} corresponds to 19.5 Hz in case 1. The rotor runout is greater than that of the stator in the region of $\omega < \omega_{rs}$ and $\omega > \bar{\omega}_{rs}$. $\bar{\omega}_{rs}$ is $\bar{\omega}_{rs} = \sqrt{(K_s - K_r)/(M_s - M_r)}$. The value of $\bar{\omega}_{rs}$ is 34.1 Hz in case 1. In case 2, if the position of the natural frequency of the rotor and stator is reversed as shown in Fig. 4, all the above comment also applies as in case 1.

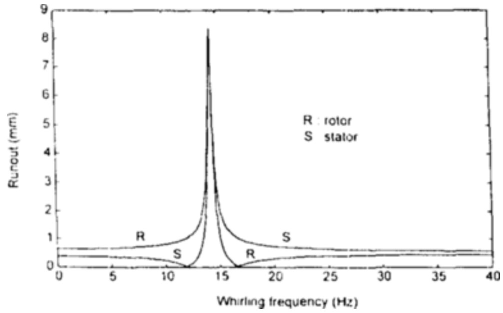

Fig. 4 Runout of rotor and stator vs. whirling frequency for case 2

4. Slipping and Rolling

The observations of operating rotor in experiment show that the foregoing explanation is not complete to understand the rubbing phenomena. Not only partial rub or backward slipping occurs but also the predicted region of backward rolling is not consistent with the foregoing predictions. Black(1968) explained backward slipping can start at the corresponding rotor speed to the combined natural frequency since at that point the ratio of the the tangential and normal force exceeds the Coulomb friction coefficient. But this explanation is not enough to explain the maximum rotor speed where backward slipping maintains. Crandall(1990) showed that the frequency of backward rolling can not exist in the region of $\omega_{rs} \leq \omega \leq \omega_o$ by stability analysis for the steady state responses of the rotor and stator. The value of ω_o is somewhat greater than the natural frequency of stator in case 1. This explanation is good for the nonexistence condition of backward rolling, but this explanation is a roundabout way. It is not enough to explain why backward slipping occurs in that region directly. And, it was not clearly verified by experiment. In this respect, another physical explanation is attempted in this study using the concept of the receptance of the rotor and stator.

Theoretical responses in the previous section correspond to equilibrium solutions. The existence of equilibrium solutions is usually checked by stability analysis. The stability analysis can be done on the basis of the physics of rotor/stator system. This rubbing problem can be modeled as a simplified form of forced vibration problem during contact as follows.

$$M_r\ddot{x}_r + K_r x_r = -N\cos\omega t$$
$$M_s\ddot{x}_s + K_s x_s = N\cos\omega t \qquad (8)$$

If the deviation exists, the following equation should be satisfied.

$$M_r\Delta\ddot{x}_r + K_r\Delta x_r = -\Delta N\cos\omega t$$
$$M_s\Delta\ddot{x}_s + K_s\Delta x_s = \Delta N\cos\omega t \qquad (9)$$

To satisfy the full annular rub condition, the deviation of the rotor runout should always be greater than that of the stator. Otherwise, the contact can not be assured. It needs to check that the magnitudes of the deviations satisfy :

$$\Delta x_r = \left|\frac{\Delta N}{K_r - M_r\omega^2}\right|$$
$$\Delta x_s = \left|\frac{\Delta N}{K_s - M_s\omega^2}\right| \qquad (10)$$

Since the deviations of the normal force on the rotor and on the stator have the same magnitude during contact, the magnitude of the deviated runout of the rotor and stator depends on the numerator of Eq. (10). It is the same form of receptance. If the receptance of the rotor is greater than the receptance of the stator at that frequency, the contact condition is assured, and, the deviation of the normal force goes to zero. The equilibrium solution becomes then a stable solution, and, the assurance of contact makes backward rolling. If $\Delta x_r < \Delta x_s$ at a whirling frequency, the contact can not be assured. The equilibrium solution is no longer a stable solution. This unstable situation goes to a stable situation or slipping will occur. It can be noted that slipping always accompanies a repetitive contact and separation.

For case 1, the dynamic stiffness, the reverse of the receptance, is shown in Fig. 5 for clarification of the intersection points. As the result, in the region of $\omega < \omega_{rs}^{+}$, backward rolling can occur where ω_{rs}^{+} is ω_{rs}. But in the region of ω_{rs}^{-}, slipping will occur, therefore, backward slipping or forward whirl will occur. The corresponding rotor speed for backward rolling will be $\Omega = C\omega/a$. In the range of rotor speed of $C\omega_r/a < \Omega < C\omega_{rs}^{-}/a$, two stable whirling speeds of ω_{rs}^{+} and ω_{rs}^{-} exist, but, the ω_{rs} is physically unstable from Eq. (3). Therefore, if backward slipping occurs, the whirling frequency will be ω_{rs}^{+} since slipping occurs.

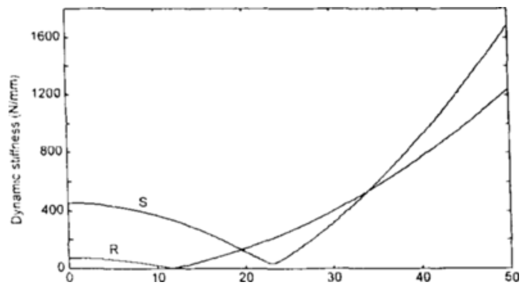

Fig. 5 Dynamic stiffness of rotor and stator for case 1

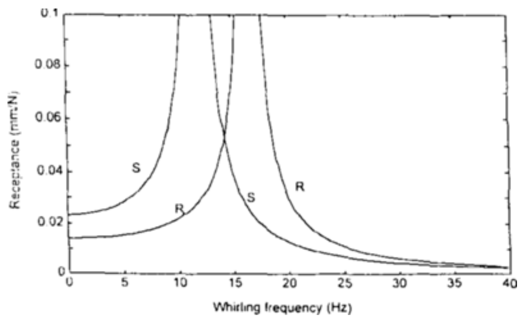

Fig. 6 Receptance of rotor and stator for case 2

Case 2 can be explained in the same way as case 1. However, there is no lower limit as like ω_{rs}^{+} of case 1 as shown in Fig. 6. Therefore, backward slipping is not possible. Instead, free rotor run or partial rubbing is more plausible. If forward whirling occurs, the forward synchronous whirl will be possible with the previously discussed positive normal force condition in the range of $\omega_s < \Omega < \omega_{rs}^{+}$.

5. Eccentricity Effect

Throughout the foregoing explanations it is not clear for which forward whirling and backward slipping will occur at a specific rotor speed. And, there is no explanation where backward slipping ends during increasing the rotor speed. This is because eccentricity is not considered, which can not be neglected in practical situations. Eccentricity is the source of rotor runout. Before the rotor contacts with the stator, free whirling motion of rotor occurs due to the induced centrifugal forces by eccentricity. When Eq. (2) and the geometric constraints of Eq. (1) are solved simultaneously, the whirling frequency is another variable. It is better to assume the type of whirling motion at the beginning.

If backward rolling is assumed in backward whirling analysis, the displacements of the rotor and stator have two components of frequencies. One is due to whirling and the other is due to rotor speed. Therefore, the phase difference between the displacement of rotor geometric center with respect to the fixed point and the displacement of rotor mass center with respect to the rotor geometric center changes from 0 to 2π throughout backward whirling process. Because dynamic force equilibrium should be satisfied at each specific phase angle, the combination of the steady state response at a given phase shift throughout 0 to 2p can be the steady state solution of backward whirling with eccentricity. Checking the sign of normal force and the ratio of tangential force to normal force can confirm the possibility of the occurrence of backward whirling.

From Eq. (2) with the assumed response of rotor and stator of Eq. (5), following complex type equations result for a specific phase difference.

$$\{(K_r - M_r\omega^2) + iC_r\omega\} r_r + N(1+i\mu)e^{i\tau} = M_r e\Omega^2 e^{i\beta}$$
$$\{(K_s - M_s\omega^2) + iC_s\omega\} r_s - N(1+i\mu)e^{i\tau} = 0 \tag{11}$$

Eqs. (6) and (11) constitute six algebraic equations with the six unknowns of r_r, r_s, N, μ, τ and γ. The results are as follows.

$$\tau = \beta_{rs} - \beta_s - \sin^{-1}\left[\frac{M_r e\Omega^2}{|R_s|C}\sin(\beta - \beta_{rs})\right]$$
$$r_r = \frac{C|R_s|}{|R_{rs}|}\cos(\tau + \beta_s - \beta_{rs}) + \frac{M_r e\Omega^2}{|R_{rs}|}\cos(\beta_s - \beta_{rs})$$
$$N = -0.5(|R_{rs}| r_r \cos(\beta_{rs} - \tau) + C|R_s|\cos\beta_s + M_r e\Omega^2\cos(\beta - \tau))$$
$$T = \mu N = -0.5(|R_{rs}| r_r \sin(\beta_{rs} - \tau) + C|R_s|\sin\beta_s + M_r e\Omega^2\sin(\beta - \tau))$$
$$\gamma = \tan^{-1}\frac{-C\sin\tau}{r_r - C\cos\tau}$$
$$r_s = [r_r^2 + C^2 - 2r_r C\cos\tau]^{1/2} \tag{12}$$

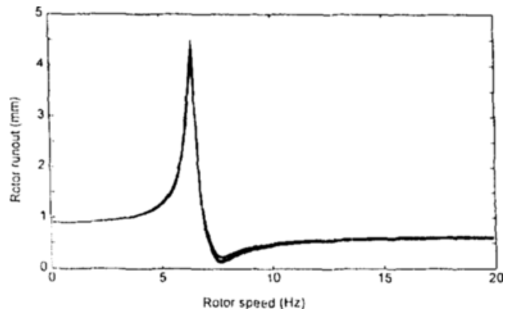

Fig. 7 Rotor runout during backward rolling with eccentricity for case 1

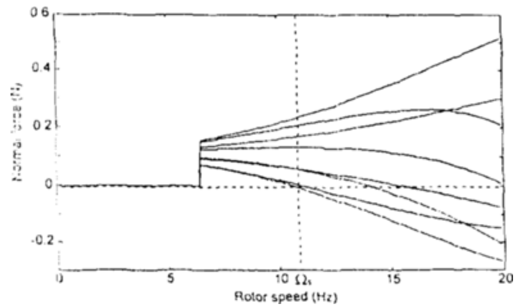

Fig. 8 Variation of normal force due to eccentricity during backward slipping for case 1

The calculated results are shown in Fig. 7 for the rotor runout for case 1 under the assumption of backward rolling. There are some deviations in the rotor runout compared to Fig. 3 due to the eccentricity effect. The deviation of rotor runout means a beat phenomenon with a whirling component and with a rotor speed component.

In the region of the predicted backward slipping, although the rotor speed changes, the whirling frequency does not change. The backward whirling frequency of ω_{rs}^{+} will be maintained during backward slipping process. As the rotor speed increases, the deviation of the normal force will be greater due to the eccentricity effect. Figure 8 for case 1 shows that as the rotor speed increases, the normal force goes to zero at a specific phase shift at a rotor speed, Ω_s which is

10.6 Hz for case 1. Above the rotor speed of the initiation of zero normal force, the rotor starts to separate from the stator. The rotor can maintain the contact to a certain extent due to the inertia effect. Otherwise the rotor will bounce due to impact. Eventually the backward slipping will stop, and the whirling pattern will be changed. Consequently, the full annular rub beyond the rotor speed cannot be guaranteed .

The whirling frequency is the same as the rotor speed in forward synchronous whirling. The Eq. (2) can be solved with a given friction coefficient value. The normal force should be positive for the full annular rub condition. There should be a specific value of phase angle between the rotor geometric and mass centers. This forward whirl can be calculated from Eqs. (5) and (11). The unknown will be β instead of μ.

6. Experimental Results and Discussion

In order to verify the foregoing physical explanations of rubbing phenomena, experiments were performed. The experimental apparatus designed by Glitzenstein(1988) was used. Details of the apparatus are shown in Fig. 9.

The Rotor consists of a 0.686 m long with 0.64 cm diameter steel shaft supported by two bearings on which flywheels are mounted. The stator is composed of a square brass mass, a replaceable aluminum plate, and four aluminum rods. The stiffness of the stator can be changed by adjusting the length of aluminum rods. Contact point on the stator with the rotor is a replaceable aluminum plate with a hole at the center.

To measure the x and y displacements of rotor, two Electro-Mike electromagnetic displacement transducers are employed. Two B & K 4344 accelerometers are also used to measure the x and y directional accelerations of the stator. The rotor speed is measured by an eddy current detector through the use of a 60-teeth gear press-fit onto the transmission shaft. These 5 measured signals are passed through low pass filters which attenuate signals over 100 Hz to eliminate high frequency noise. The rotor runout signals through proximeters are monitored by a Hewlett Packard 54600A oscilloscope. All the signals enter the computer using LWB(1990) for each rotor speed. The recorded signals were then analyzed by MATLAB. Eccentricity was measured through the rotor runout on the running conditions of the rotor without the stator for different rotor speeds. The eccentricity of rotor was 0.5 mm. Experiments were done for the previously mentioned two cases.

6.1 CASE 1

In order to achieve the greater natural frequency of the stator than that of the rotor, two flywheels are attached to the shaft for the purpose of reducing the natural frequency of the rotor, and a brass plate is used as the stator. The length of aluminum rod is reduced to 8.5 cm for higher stator natural frequency. The experimental results are shown in Figs. 10 and 11.

Figure 10 shows the whirling frequency vs. the

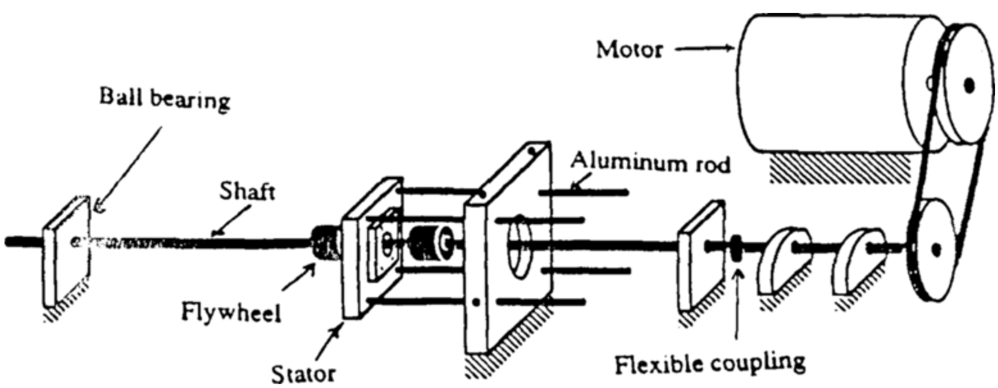

Fig. 9 Experimental apparatus

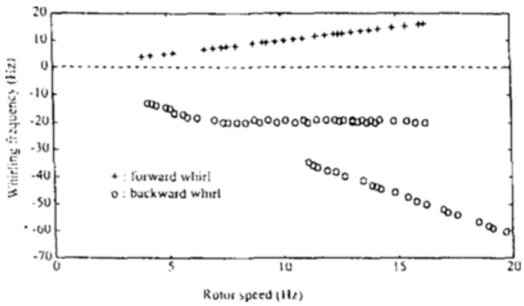

Fig. 10 Whirling frequency vs. rotor speed in case I by experiment

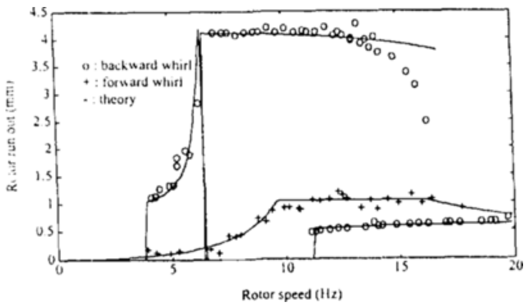

Fig. 11 Comparison of the results of experiment and theory for case I

rotor speed. Free rotor whirling of no rub condition is detected in the range of the rotor speed of $\Omega < 9.7$ Hz or $\Omega > 16.3$ Hz, as shown in Fig. 11. In the rotor speed of $9.7 \text{ Hz} < \Omega < 16.3$ Hz forward whirl can occur, but the receptance of the rotor is greater than that of the stator in those regions. Accordingly, there is no chances to slip, which is inevitable for the occurrence of forward whirling. Instead, the whirling motion becomes a partial rub.

In backward whirling, the predicted rotor speed of backward rolling starting is 3.86 Hz. The experimental results show that backward whirling starts at the rotor speed of 4.1 Hz. The discrepancy of the data is because the experimental apparatus cannot maintain a constant speed of rotor at that low speed range of operation. As the rotor speed increases, backward slipping starts in the vicinity of 6.4 Hz, the corresponding rotor speed of ω_{rs}^{*}, 19.5 Hz as was predicted. The backward slipping ends at 16.2 Hz of the rotor speed. This rotor speed can be obtained with the very slow increasing rotor. In most cases the backward slipping ends at the rotor speed less than 16.2 Hz. The maximum rotor speed with backward slipping depends on the increasing rate of rotor speed or a stable operation of rotor. This means that partial rub starts at that rotor speed, and eventually the prevalence of partial rub breaks the backward slipping. This is because due to the eccentricity effect, as was predicted. The value of Ω_{slip} in Fig. 7 is 10.6 Hz in this case. If the rotor speed decreases from high speed with backward rolling, the backward whirling frequency continues until 11.2 Hz of rotor speed which corresponds to ω_{rs}. The backward whirling frequency jumps abruptly to ω_{rs}^{*} as was predicted since the contact is not so firm. The runout of backward whirling in the region of greater than ω_{rs}^{*} is small. But, once backward slipping happens the runout becomes abruptly large as shown in Fig. 11.

6.2 CASE 2

To decrease the natural frequency of the stator, an aluminum plate was used and long aluminum rod of 20.6 cm is used. The experimental results are shown in Figs. 12, 13. In this experiment backward whirling and forward whirling both appeared. The forward whirling response starts to appear near $\omega_s = 11.9$ Hz, and continues until $\omega_{rs}^{*} = 14.3$ Hz. In the forward whirl case, from $\omega_s = 11.9$ Hz to $\omega_{rs}^{*} = 14.3$ Hz, the receptance of the stator is greater than that of the rotor. As a result, there is a possibility to slip, which can make the forward whirl. Backward whirling starts at 7 Hz. As the rotor speed increases, forward whirling components are more prevalent, eventually resulting in partial rub. The runout due to forward whirling is very large, but the runout due to backward whirl is relatively small.

In case of backward whirling, though the stator receptance is larger in the region of less than $C\omega_{rs}^{*}/\alpha$ of 3.8 Hz, there is no lower limit where the receptance of the rotor and stator coincide. Therefore, backward slipping cannot happen. Backward rolling can take place from ω_r to ω_{rs}^{*}. But in this case there is no real value of ω_{rs}^{*}. Therefore, no backward slipping can be expected. But, due to the eccentricity effect, full annular backward roll-

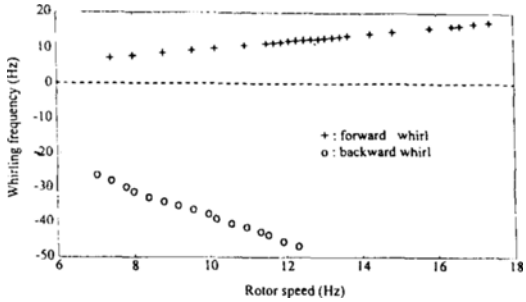

Fig. 12 Whirling frequency of case 2 by experiment

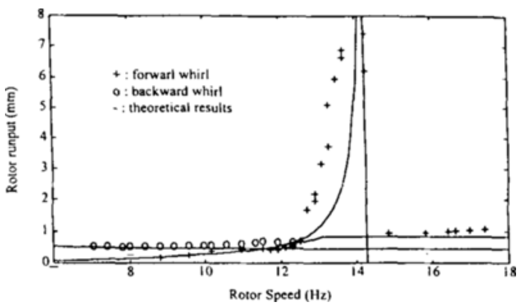

Fig. 13 Comparison of the results of experiment and theory for case 2

ing can occur in low speed range. As the rotor speed increases, a partial rub will appear. The rotor speed of 4.3 Hz corresponds to ω_{rs}. But, because of low speed and small clearance, backward rolling is observed from 7 Hz.

7. Conclusion

In order to investigate destructive rubbing phenomena in rotor dynamics, a new approach was done in this study. Theoretical model was assumed to be linear, axisymmetric, and perfect alignment of the rotor and stator. The theoretical analysis was done for cases of full annular rub throughout whirling motion. Elastic stator case without the consideration of eccentricity shows that rubbing phenomena can be categorized as the case 1 of $\omega_s > \omega_r$ and case 2 of $\omega_r > \omega_s$. The responses of rotor/stator system using complex notation results that there is no full annular rub in the range of $\omega < \omega_r$ and $\omega_{rs}^{+} < \omega < \omega_s$ for the case 1, and in the range of $\omega < \omega_s$ and $\omega_{rs}^{+} < \omega < \omega_r$ for case 2. The receptance consideration shows that rotor can slip in the range of $\omega_{rs}^{+} < \omega < \omega_{rs}^{-}$ in case 1 and in the range of $\omega < \omega_{rs}^{+}$ in case 2. In that range, backward slipping or forward whirling is possible. The consideration of eccentricity of rotor shows that if backward rolling occurs, there is a small deviation of runout, which results in a beat phenomenon and thicker orbit. If backward slipping occurs, the eccentricity effect can break the full annular rub condition as rotor speed increases, and if forward whirling occurs, there should be a specific phase difference between the displacement of rotor center and that of rotor geometric center. Theoretical predictions were also confirmed by the experiments for two cases. The whirling frequencies and runout were in good agreement with the theoretical predictions.

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